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LETTER TO THE EDITOR

Simplified calculation of the relaxation of the temperature anisotropy in a plasma due to binary encounters

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Abstract. A test particle is selected to characterize the average behaviour of an assembly of particles of the same kind in a plasma. The re-distribution of energy of this particle after binary interaction between it and the other members of the assembly is calculated.

A comparison is made between the simple *physical* solution offered here and that of Kogan. The present result can be easily applied to plasma problems, for example, the solar wind.

The relaxation of the temperature anisotropy in a plasma has been investigated theoretically in recent years and applied practically in research on controlled fusion. Relaxation takes place by processes which are regarded as separable, namely, binary encounters between charged particles and collective effects. Kogan (1961) was the first to solve the problem for the equalization of temperature components of particles of the same kind as brought about by binary encounters amongst themselves. An extension of this work was provided by Lehner (1967). Here, the object was to treat the same problem as that of Kogan but in a simplified physical manner.

Let the thermal velocity distribution of the assembly of particles be isotropic in a plane orthogonally transverse to the z direction. This direction is taken as the direction of the magnetic field should it exist. Select one particle from the assembly with velocity v and regard it as a test particle. Impose the restriction that the velocity components v_t and v_z of this particle in the respective transverse direction and z direction be equal to the RMS values of speed, $\tilde{v}_t$ and $\tilde{v}_z$. If θ is defined as the angle between the z direction and the direction of the velocity v,

$$\sin^2\theta = \frac{\tilde{v}_t^2}{\tilde{v}_z^2+\tilde{v}_t^2} = \frac{2}{K+2}$$

and (1)

$$\cos^2\theta = \frac{\tilde{v}_z^2}{\tilde{v}_z^2+\tilde{v}_t^2} = \frac{K}{K+2},$$

where K is the temperature-anisotropy factor, that is, the ratio of the kinetic temperatures in the respective z direction and transverse direction. All particles which satisfy the speed condition given above are characterized by the fact that their velocity vectors form the envelope of a double cone as illustrated in figure 1. The test particle thus

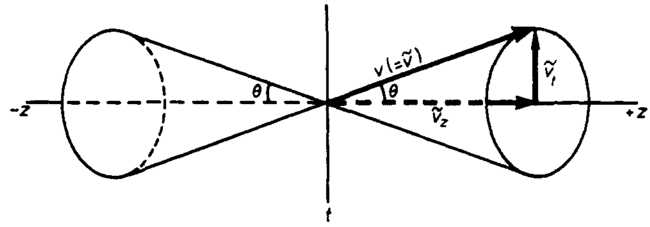

Figure 1. Velocity diagram of particles possessing the same velocity as the test particle, before interaction with the assembly of particles.

described is to be regarded as representing the average behaviour of the assembly of particles. The problem is to calculate the re-distribution of energy of this particle after binary interaction between it and the other particles of the assembly.

Let $\Delta v_\perp$ be the change of speed per unit time in the direction perpendicular to the original motion of the test particle caused by the single interaction between it and the remainder of the assembly. The mean value of $(\Delta v_\perp)^2$ over many such interactions is

$$\langle(\Delta v_\perp)^2\rangle = \frac{0{\cdot}7\times 8\pi n(ze)^4 \ln\Lambda}{m^2\tilde{v}}, \tag{2}$$

where n is the number density of the particles, m and ze the respective mass and electric charge of a particle, Λ the ratio of the Debye–Hückel radius to the mean impact parameter and $\tilde{v}$ is the RMS speed. This result is based on equation (5.17) of Spitzer (1962) with the assumption that this equation is not seriously affected in the case where the velocity distribution is anisotropic. The average amount of energy $\epsilon_\perp$ transferred per unit time in the direction perpendicular to the initial direction is $\frac{1}{2}m\langle(\Delta v_\perp)^2\rangle$.

If one assumes a *statistically* isotropic distribution of the energy $\epsilon_\perp$, then the velocity vector associated with this energy describes a circle in a plane which is perpendicular to the direction of the velocity v (see figure 2). Choose two directions in this plane, one in the plane of figure 2 and the other in the direction perpendicular to the plane of this figure, that is, the transverse direction. The energy in each direction is on the average $\epsilon_\perp/2$. In the plane of figure 2, the energies in the z direction and transverse direction are respectively $\frac{1}{2}\epsilon_\perp \sin^2\theta$ and $\frac{1}{2}\epsilon_\perp \cos^2\theta$. Hence, on using equation (1), the components

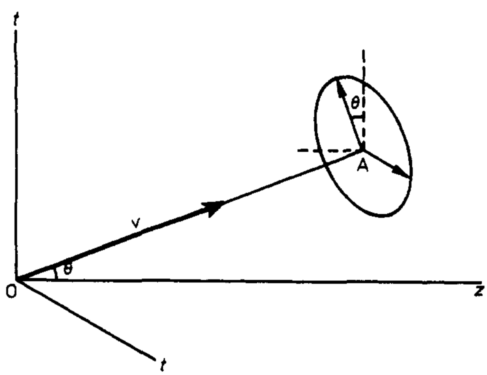

Figure 2. Velocity diagram of the test particle illustrating the re-distribution of velocities after interaction with the assembly of particles. OA is the direction of the initial motion of the test particle with velocity v before the interaction with the other particles.

of transferred energy are $\epsilon_\perp/(K+2)$ in the z direction and $[\epsilon_\perp K/2(K+2)]+\epsilon_\perp/2$ in the transverse direction. The energy of the particle in the initial direction after the interaction is depleted by the amount $\epsilon_\perp$. In the z direction this amounts to $\epsilon_\perp \cos^2\theta$ which is equal to $\epsilon_\perp K/(K+2)$, whilst in the transverse direction it is $\epsilon_\perp \sin^2\theta$ which is equal to $2\epsilon_\perp/(K+2)$.

Denote by ϵ_z and ϵ_t the total respective changes in energy per unit time in the z direction and transverse direction. Hence, on collecting terms,

$$\epsilon_z = \frac{\epsilon_\perp}{K+2} - \frac{\epsilon_\perp K}{K+2} = -\left(\frac{K-1}{K+2}\right)\epsilon_\perp \tag{3}$$

and

$$\epsilon_t = \frac{\epsilon_\perp K}{2(K+2)} + \frac{\epsilon_\perp}{2} - \frac{2\epsilon_\perp}{K+2} = \left(\frac{K-1}{K+2}\right)\epsilon_\perp. \tag{4}$$

The result $\epsilon_z = -\epsilon_t$ is as anticipated. Now on combining equations (3) and (4) with equation (2),

$$\epsilon_t = -\epsilon_z = \frac{0\cdot 7 \times 4\pi n (ze)^4 \ln \Lambda}{(3mkT)^{1/2}}\left(\frac{K-1}{K+2}\right), \tag{5}$$

where T is the mean kinetic temperature and k the Boltzmann constant. This is the final result.

The result given by equation (5) can be compared with the results of either Kogan (1961) or Lehner (1967). The former regarded his results as accurate only to an order of magnitude. Their results can be reduced to the formulation given here by using the equations

$$T_t = [3/(K+2)]T \qquad \text{and} \qquad T_z = [3K/(K+2)]T.$$

One obtains

$$\epsilon_t = \frac{2\pi^{1/2} n (ze)^4 \ln \Lambda}{(mkT)^{1/2}}\left(\frac{K}{K-1}\right)\left(\frac{K+2}{3K}\right)^{1/2}\left[\left(3-\frac{K-1}{K}\right)\phi\left(\frac{1-K}{K}\right)-3\right] \tag{6}$$

where the function

$$\phi(x) = \begin{cases} \dfrac{\tan^{-1} x^{1/2}}{x^{1/2}} & \text{when } x>0 \\[2ex] \dfrac{\ln\{[1+(-x)^{1/2}]/[1-(-x)^{1/2}]\}}{2(-x)^{1/2}} & \text{when } x<0. \end{cases}$$

Over a wide range of values of K, for example, from zero to 100, including those typical of the solar wind, equations (5) and (6) compare favourably and differ by a factor of up to 2.

Equation (5) was used in calculating the effect of binary encounters on the thermal anisotropy of the protons in the solar wind (Eyni and Kaufman 1975). It may be inferred from the results of that paper (Eyni and Kaufman, p 111) that the collisional effect as represented by equation (5) stands between Kogan's (1961) result on the one hand and that of Leer and Axford (1972) on the other.

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